Lambda Calculus Homework 1

Due one week from time assigned

\*\*Deadline is FIRM as sample solutions will be available afterwards\*\*

Write your solutions clearly by hand and submit scans (in pdf) or

clear photos of your solutions, or submit hard copy at end of class.

0. Identify the free and bound variables in the following term:

lambda u. lambda x. [y (lambda y.x y) u y]

Remember: whether a term is free or bound is always relative to a given

term. What are the free and bound variables for just the term inside the

[]'s ?

1. Determine if the following pairs of terms are alpha-equivalent.

a. lambda x.lambda y. (x y x) and

lambda x.lambda y. (y x y)

b. lambda x. lambda y. x (lambda y.x) y and

lambda a. lambda b. a (lambda u.a) b

For the rest of the problems, find the beta-normal forms of the

following terms. Remember: you can rename bound variables to

avoid possible clash, but never free variables. Also, application

associates to the left: a b c is the same as (a b) c, but not the

same as a (b c). Also, do not confuse lambda x.x y with (lambda x.x) y:

application has precedence over abstraction.

2. (lambda x.lambda y. (y x)) u (lambda x.x)

3. given S, K, I as they are in the handout, find the normal forms

of KII, SIK, SK(KI), and S(KI). Application associates to the left,

and the order is important. Also remember that application binds

tighter than lambda-abstraction, so (lambda x.x y) is NOT the same

as (lambda x.x) y.

4. given A = lambda m.lambda n.lambda f.lambda x. m f (n f x),

T = lambda m.lambda n.lambda f.lambda x. m (n f) x

Zero = lambda f.lambda x.x

One = lambda f.lambda x.(f x)

Two = lambda f.lambda x.f (f x)

Find the normal form of (A Two One).

Then find the normal form of (T Zero Two)

What do A and T stand for?

4b. (NEW - OPTIONAL CHALLENGE)

Church exponentiation, m to the nth power, is lambda m.lambda n.(n m).

Show that 3 to 2rd power = 9 using the Church representation.

5. Given T = lambda x. lambda y. x (same as K)

F = lambda x. lambda y. y (same as KI)

D = lambda p.lambda q. (p T q) (where F is as above)

Find the normal forms of:

a. (D F F)

b. (D F T)

c. (D T F)

d. (D T T)

Does anything about the behavior of these lambda terms look familiar?

(pretend you're all excited now and want to know more!)

6. Let T and F be as they were in the above problem, and let

N = lambda x. (x F T)

what are the normal forms of

a. (N F)

b. (N T)

c. (N (N T))

( he he he ... )

7. Let T and F be as above and let IF = lambda c.lambda a.lambda b.(c a b).

Find the normal forms of

(IF T A B)

(IF F A B)

For arbitrary terms A, B (what A and B are shouldn't be important).

8. let PAIR = lambda a. lambda b. lambda c. (c a b)

let FST = lambda p. p T (where T is as above)

let SND = lambda p. p F

let M = (PAIR a (PAIR b c))

Find the normal forms of (FST M), (SND M) and (SND (SND M))

9. (challenge). Design a pure lambda term ISZERO that determines if

a church numeral is zero, that is:

ISZERO ZERO should beta-reduce to TRUE (K), but for non-zero values:

ISZERO (lambda f.lambda x.f (f x)) should beta-reduce to FALSE.

Show that these reductions indeed hold for your ISZERO.

10. (not recommended unless you have a lot of time..) find the normal form of

(lambda x. x x) (lambda x. x x)